Problem 4. Moral Hazard

Problem statement

The architect Ivanov discovered the photos of houses he designed on a website which was not his. He ran an investigation and learned that a less experienced architect Sidorov used Ivanov's ideas for his own purposes. Ivanov decided to sue Sidorov and hired a lawyer specializing in intellectual property. It is known that the utility function of the lawyer is U (w ,e) = $\sqrt{w}$ – 2e, where w is his wage, and e is related efforts.

The lawyer has to prove that actions of Sidorov caused negative effects on Ivanov. It is easy to prove Sidorov guilty as long as the lawyer puts minimal necessary effort to the case (e=1). The situation is, however, different concerning the compensation that Ivanov may receive. If the lawyer puts low effort (e=1), a judge will probably charge quite a low compensation of 100 monetary units. If the lawyer puts significantly more effort (e=5), the compensation might reach 1000 monetary units.

The lawyer in question is quite popular, so he can take some other case and gain an alternative value $\overline{U} = 6$. It is known that the lawyer will prefer to work with the case of the architect if another customer offers the same fee.

Legislature is known to be imperfect in Russia, so even in case the lawyer puts significant effort, there is no full guarantee of success. Connection of the architect's compensation (R) with the lawyer's effort is described by the probability matrix below.

Effort	**Compensation (monetary units)**	
	R=100	R=1000
e=1	0,75	0,25
e=5	0,25	0,75

Task

1. Draw on axes w – U graphs on the architect's and the lawyer's utility. What is their relation to risk?
2. Draw the game tree.
3. What kind of a contract would the architect offer in case lawyer's efforts are observable? What utility will the lawyer have if he follows the terms of this contract? Set the expected utility of the lawyer and profit of the architect. How is the risk distributed in this contact?
4. If court decides to set a high compensation (R=1000), there is a probability *a* that the architect Sidorov will take some actions to avoid paying it at all (for instance, he will pass of his assets to his wife and divorce her). But it is known that Sidorov will not try to escape the law if compensation is rather low (R=100).

   What should the probability *a* be like to affect the choice of contract by the architect in case there are <u>observable</u> efforts? Find whether there is *a* under which the architect should not hire the lawyer at all.
5. In a real-life situation it is very hard to observe efforts of the lawyer, so the architect has to stem from the idea that it is impossible to control actions of the lawyer. Count the expected income of the architect and the expected lawyer's fee for this type of contract.
6. It is known that the architect will never agree to give more than a half of his or her compensation to the lawyer as he thinks that this way he will let the lawyer to earn unfair money. How will the architect act in this case? Find if it is profitable for the architect to hire the lawyer,

and if yes, what contracts will he offer. How will it affect the expected utility of the lawyer and the expected profit of the architect?

Solution

1. *Utility graphs and attitude to risk*

For the utility function U (w ,e) = $\sqrt{w}$ – 2e, it is true that

U'e = -2 <0,

so with an increase of efforts of the lawyer, his or her utility grows.

U'w = 1/(2$\sqrt{w}$) >0. U''w^2= - 1/(4w$\sqrt{w}$) < 0

With an increase of a fee of the lawyer, his or her utility grows, and the rate of increase falls. Thus, the lawyer has negative attitude to risk.

The graph is standard, the function is concave.

The architect is neutral to risk, as he tends to maximize the profit. The graph is a straight line.

2. The game tree is standard.

3. *The case of observable efforts.*
Let us count the equilibrium fee X. The lawyer will agree to work for the architect only if U (w,e) ≥ $\overline{\text{U}}$

so $\sqrt{w}$ – 2e ≥6

If e= 1, than m = **64.**
If e = 5, than X = **256.**

The incentive contract will be
$$w=\begin{cases}256 \text{ if } e=5\\ m2 \text{ if } e=1\end{cases}$$

m2 < m, so that the lawyer does not have incentives to choose a contract with low efforts.

The expected profit of the principal is:
ER = 0,25 * 100 + 0,75 * 1000 = **775**
The net profit of the principal is:
ER_{NET} = 775-256 =519
The utility of an agent is:
U = $\sqrt{256}$ – 2*5 = 16- 10 = 6 (equal to an alternative utility)

The architect will offer a fee w = 256 to the lawyer. Is it beneficial?

Let us agree that the architect wants to stimulate a minimal level of the lawyer's effort (e=1).
$\sqrt{w}$ – 2*1 ≥6
w = **64**

$$w = \begin{cases} X2 \text{ if } e = 5 \\ 64 \text{ if } e = 1 \end{cases}$$

X2< X, so that the lawyer does not have a reason to choose contract presupposing more effort.

Costs of extra effort of the lawyer are 256-64 = 192.

The expected profit of the principal is: ER = 0,75 * 100 + 0,25 * 1000 = 325
The expected net profit of the principal is: ER_{NET} = 325-64 = 261
Extra effort of the lawyer guarantee the increase of the expected profit to 519 – 261 = 258.
The expected utility of the lawyer is U = 8 – 2 = 6.
Consequently, the architect is in a position where it is advantageous if the lawyer puts more effort.

If the effort of the lawyer is observable, the expected fee fully matches the fee stated in a contract. The lawyer does not bear any risks.

4. *Non-obedience to the decision of the court.*
As the work of the lawyer is unobservable, the new condition will not influence the sums of contracts on a different level of effort. However, it might influence the choice of the architect concerning the lawyer (if it is advantageous to stimulate the lawyer or not).

The probability *a* lowers the expected income of the high compensation (if R=1000):
ER = 0,25 * 100 + 0,75 (1-a) * 1000 = 25 +750 – 750a = 775 – 750a

and consequently, the expected net profit is
ER_{NET} = 519 – 750a

Let us consider the condition under which it is useless to make an incentive contract with the lawyer:

519 – 750a $\leq$ 0

750a $\geq$ 519

If a < 0,692, the expected profit remains positive if the architect stimulates the efforts of the lawyer (e=5). However, it is worth considering the option of low effort.

If e = 1, the expected compensation gets lower as well:

ER = 0,75 * 100 + 0,25* (1-a) * 1000 + a*0 = 325 – 250a.

- together with net profit:

$ER_{NET =}$ 261 – 250a.

Let us consider conditions under which it will make sense:

261 - 250a $\geq$ 0

250a $\leq$ 261

a $\leq$1,044.

As a cannot me higher than 100%, there does not exist a kind of an *a* under which the expected profit is equal to 0 or is negative provided that the effort of the lawyer is minimal. It means that the architect is in advantage from going to the court in any case.

Let us consider a condition, under which it is not advantageous for the architect to create incentives for the lawyer.

519 – 750a < 261 – 250a

(let us read the equality as a reason to avoid extra effort)

500a > 258

a > 0,516

Thus, there is a following incentive contract:

$$w=\begin{cases} 256, e = 5 \text{ if } a < 0{,}516 \\ 64, e = 1 \text{ if } 1 > a \geq 0{,}516 \end{cases}$$

Result: if the probability that the defendant will choose to avoid legal obligations is higher than 51,6%, the architect must not introduce incentives to the work of the lawyer. However, it is profitable to go to court in any case.

5. *Let us analyze the situation of unobservable efforts.*

Let us find the contract

$$w=\begin{cases} x \text{ if } R = 1000 \\ y \text{ if } R = 100 \end{cases}.$$

The following set of inequalities must be fulfilled.

$EU_{e=5} \geq \bar{U}$ (individual rationality constraint)

$EU_{e=5} \geq EU_{e=1}$ (incentive compatibility constraint)

The expected utility of the lawyer in case he works very hard is:
$EU_{e=5} = 0{,}25 * (\sqrt{x} - 10) + 0{,}75 * (\sqrt{y} - 10)$.

The individual rationality constraint is:
$0{,}25 * (\sqrt{x} - 10) + 0{,}75 * (\sqrt{y} - 10) \geq 6$
$0{,}25\sqrt{x} – 2{,}5 + 0{,}75\sqrt{y} – 7{,}5 \geq 6$

$0{,}75\sqrt{y} \geq 16 - 0{,}25\sqrt{x}$
$\sqrt{y} \geq 21\frac{1}{3} – \frac{1}{3}\sqrt{x}$

The incentive compatibility constraint is:
$0{,}25 * (\sqrt{x} - 10) + 0{,}75 * (\sqrt{y} - 10) \geq 0{,}75 * (\sqrt{x} - 2) + 0{,}25 * (\sqrt{y} - 2)$
$0{,}25\sqrt{x} – 2{,}5 + 0{,}75\sqrt{y} – 7{,}5 \geq 0{,}75\sqrt{x} - 1{,}5 + 0{,}25\sqrt{y} – 0{,}5$
$0{,}5\sqrt{y} \geq 8 + 0{,}5\sqrt{x}$

$\sqrt{y} \geq 16 + \sqrt{x}$

If e = 5,
ER_{NET} = 0,25 (100 – x) + 0,75 (1000 – y) = 775 – 0,25x – 0,75y

If $\sqrt{y}$ = b, $\sqrt{x}$ = a, then the optimal contract is a solution to the optimization task:
max (775 - 0,25a^2 – 0,75b^2)
s.e.
b ≥ 21 $\frac{1}{3}$ - $\frac{1}{3}$ a
b ≥ 16 + a,
- or it it the same as:
min (0,25a^2 + 0,75b^2)
s.e. b ≥ 21 $\frac{1}{3}$ - $\frac{1}{3}$ a
b ≥ 16 + a

The minimized function can be drawn as an ellipse along the $\sqrt{x}$ axis with a center in a point (0,0). The optimum for this function on the given set will be either the contact point of the level of the given ellipse with the line $\sqrt{y} \geq 21\ \frac{1}{3} - \frac{1}{3}$, or the point of the corner extremum defined as the point where the individual rationality constraint and the incentive compatibility constraint meet.
Let us first consider the last variant. Solving a set of equations

$\sqrt{y} = 21\frac{1}{3} - \frac{1}{3}\sqrt{x}$

$\sqrt{y} = 16 + \sqrt{x}$,

we get

$\boldsymbol{\sqrt{y}}$ **= 20;** $\boldsymbol{\sqrt{x}}$ **= 4**.

Let us now consider the first case. The task of conditional optimisation becomes

Min (0,25a^2 + 0,75b^2)

$b = 21\frac{1}{3} - \frac{1}{3}a.$

Let us solve it with the method of Lagrange multipliers.

Lagrangian: $\mathcal{L} = 0{,}25a^2 + 0{,}75b^2 + \lambda\ (b - 21\frac{1}{3} + \frac{1}{3}a)$

First order condition:
$\mathcal{L}'$a = 0,5 + $\frac{1}{3}\ \lambda$
$\mathcal{L}'$b = 1,5b + λ
$\mathcal{L}'\ \lambda = b - 21\frac{1}{3} + \frac{1}{3}a$ -> $\begin{cases} a = 16 \\ b = 16 \end{cases}$
The contact point lies outside the field of optimal contracts.

As we found that $\sqrt{\boldsymbol{y}}$ **= 20;** $\sqrt{\boldsymbol{x}}$ **= 4**, the optimal contract is:

$$w = \begin{cases} 400\ if\ R = 1000 \\ 16\ if\ R = 100 \end{cases}.$$

The expected profit of the lawyer is:

Ew = 0,25* 16 + 0,75*400 = 4+300 =304.

The expected profit of the architect is:

ER = 0,25* 100 + 0,75*1000 =775.

Net profit is:

ER_{NET}= 775- 304 = **471**.

The expected utility of an agent is:

EU1 = 0,75* (20 – 10) + 0,25* (4-10)= 7,5 - 1,5 = 6.

Let us count the profit of the architect in case effort of the lawyer is minimal. The utility level of the lawyer must be equal to an alternative.

Thus, the following contract will be offered:

$$w = \begin{cases} X2 \text{ if } R = 1000 \\ 64 \text{ if } R = 100 \end{cases},$$

ER = 0,25* 1000 + 0,75*100 =325,

ER_{NET} = 325 – 64 = 261,

EU2 = 0,25* (8 – 2) + 0,75* (8-2) = 1,5 + 4,5 = 6.

The incentive contract is proved to be a right decision as (471 >261).

6. As there is a condition $w \leq 0{,}5ER_{NET}$, the lawyer will be offered an incentive contract (304<327).