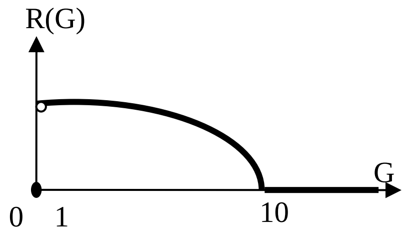
**Problem 6. Collective property**

## Problem Statement

There are 3 identical casino owners and $Y$ gamblers who love casino in a city M. Casino owners decide how many casino houses they will build. An income of each casino house directly depends on how many casino houses ($G$) are there in the city.

Opening and maintaining costs for one casino house are $c$.

$$R(G) = \begin{cases} 0, & G = 0 \\ Y - G^2, & 0 < G < 10 \\ 0, & G \geq 10 \end{cases} \cdot$$




## Task

1. Let us assume that the casino owners make decisions independently. Find how many casinos will be opened in this case. Find profit of each owner and their total profit if $Y = 100$, $c = 40$.
2. Let us assume that the owners of casinos discuss how many casinos they want to open together. How many casinos will be opened in this case? What profit will the owners have? Is it profitable to unite?
3. Compare the results of the first and the second case. Is there a problem of collective property? Please explain your answer.
4. Let us assume that while making agreements on collective actions, casino owners decide to punish any player who deviates from the contract. What minimal fine should they set in this case?
5. What will happen if owners of two casinos decide to create an alliance, and one of the owners will not accept this alliance? Will this alliance be strong? Note that the alliance and the owner of a casino that is not in this alliance make decisions on how many casinos they should open independently. Any income within the alliance is shared in half.

## Solution

**1.** If the owners of the casino houses make decisions independently, each of them will maximize his or her profit:

$$\pi_i = g_i\left(100 - \left(\sum g_i\right)^2 - c\right) \to \max_{g_i = 1,2,3}$$

$$\frac{\partial \pi_i}{g_i} = 100 - \left(\sum g_i\right)^2 - c - 2g_i\left(\sum g_i\right) = 0$$

$$g_i = \frac{100 - c - G^2}{2G}$$

Total number of casino houses will be equal to:

$$G = 3 \cdot g_1 = \frac{3(100 - c - G^2)}{2G}$$

$$2G^2 = 3(100 - c - G^2)$$

$$5G^2 = 3(100 - c)$$

$$G = \sqrt{\frac{3(100 - c)}{5}} = \sqrt{\frac{3(100 - 40)}{5}} = 6$$

Hence, each owner will open 2 casino houses and receive the following profit:

$$\pi_i = 2(100 - 36 - 40) = 48$$

Total profit: $\sum_{i=1}^{3} \pi_i = 144$.

**2.** If casino owners make a decision on opening new casino houses together, they will maximize their total profit:

$$\pi = G(100 - G^2 - c) \to \max_G$$

$$\frac{\partial \pi}{\partial G} = 100 - 3G^2 - c = 0$$

$$G = \sqrt{\frac{100 - c}{3}} = \sqrt{\frac{100 - 40}{3}} = 2\sqrt{5} \approx 4{,}47$$

$$g_i = \sqrt{\frac{100 - c}{27}} = \sqrt{\frac{100 - 40}{27}} = \frac{2}{3}\sqrt{5} \approx 1{,}49$$

Total profit is $\pi = 2\sqrt{5}(100 - 40 - 20) = 80\sqrt{5} \approx 178{,}89$, so each owner gets $\pi_i = \frac{80}{3}\sqrt{5} \approx 59{,}63$.

Comparing profit in two cases (when a decision on opening casino houses is made individually by each owner and collectively) we get that collective maximization of profit is the best strategy.

**3.** There is a problem of common property in such kind of interactions. Participants of the game have to maximize their profit within restricted space (the number of gamblers is constant). The more casinos there are in a city, the less the profit of game participants is (see table 1).

**Table 1**

| **Criteria** | **Collective decision** | **Sign** | **Individual decision** |
|---|---|---|---|
| Total number of casino houses in the city, (G) | 4,47 | < | 6 |
| The number of casinos opened by each participant, ($g_i$) | 1,49 | < | 2 |
| Total common profit, ($\pi$) | 178,89 | > | 144 |
| Each casino owner's profit, ($\pi_i$) | 59,63 | > | 48 |

Table 1 shows that there are more casinos opened if the owners make decisions independently. However, they get maximum total profit (as well as each owner's maximum profit) if decisions are made collectively. Such non-optimal allocation of resources in case of independent decision-making is related to each casino owner's desire to maximize his own profit by increasing the number of his own casinos disregarding exogenous negative effect.

**4.** Let us find a profit of a casino owner who decided to cheat on everyone else (let it be participant 1) by acting with no consideration of a collective agreement. Each owner maximizes his profit treating the number of casinos opened by the other owners as fixed:

$$\pi_1 = g_1(100 - (g_1 + g_2^* + g_3^*)^2 - c) \rightarrow \max_{g_1}$$

From p.2 we get: $g_2^* = g_3^* = \frac{2}{3}\sqrt{5} \approx 1{,}49$.

Going further on: $\frac{\partial \pi_1}{\partial g_1} = 100 - (g_1 + \frac{4}{3}\sqrt{5})^2 - 2g_1(g_1 + \frac{4}{3}\sqrt{5}) - 40 = 0$

We find $g_1$ and make a quadratic equation

$$g_1^2 + \frac{16}{9}\sqrt{5}g_1 - \frac{460}{27} = 0,$$

the roots of which are equal to 2,59 and (-6,59). As we are only interested in positive roots, the solution to the maximization task is the following: 2,59 casinos are opened by the cheating owner. And in this case his or her profit will be the following:

$$\pi_1 = 2{,}59(100 - 40 - (2{,}59 + 2{,}98)^2) = 75{,}05.$$

The profit of other owners will be:

$$\pi_{2,3} = 1{,}49(100 - 40 - (2{,}59 + 2{,}98)^2) = 43{,}17.$$

We observe that their profit is lower than the profit they expected when they agreed to cooperate. Moreover, this profit is even lower that the one they could get in the case they acted independently. Therefore, the system will come to a state of equilibrium where each owner decides independently on the number of casinos.

**5.** The minimum fine must be set in the way that it would be too costly to deviate from the cooperative strategy. It means that it should be equal to a difference between profits in cases of cooperation and cheating: 59,63 – 75,05 = - 15,37. The players would not choose to deviate from cooperation in this case.

**6.** Let us assume that the owners 1 and 2 decided to form an alliance: $g_1 + g_2 = g_4$.

The task of profit maximization in case of independent decision making will look the following way:

$$\pi_i = g_i\left(100 - \left(\sum g_i\right)^2 - c\right) \rightarrow \max_{g_i = 3,4}$$

$$\frac{\partial \pi_i}{g_i} = 100 - \left(\sum g_i\right)^2 - c - 2g_i\left(\sum g_i\right) = 0$$

$$g_i = \frac{100 - c - G^2}{2G}$$

The total number of casinos in this case will be:

$$G = 2 \cdot g_4 = \frac{(100 - c - G^2)}{G}$$

$$G^2 = 100 - c - G^2$$

$$2G^2 = 100 - c$$

$$G = \sqrt{\frac{100 - c}{2}} = \sqrt{\frac{100 - 40}{2}} = \sqrt{30}$$

The alliance and the independent owner open $0{,}5\sqrt{30}$ casinos, which bring the total collective profit $\pi = \sqrt{30}(100 - 40 - 30) = 30\sqrt{30},$ individual profit $15\sqrt{30} \approx 82{,}14$. The profit within the alliance is shared equally, so each member of it gets $7{,}5\sqrt{30} \approx 41{,}07$, which is less than the profit they could get in case of individual decisions. In other words, members of this alliance always have an incentive to leave it, so the alliance is unstable.