Problem 3. Adverse Selection

Problem Statement

Risk-neutral uncle Vanya hires grandmothers (babushkas) to bake cookies and cakes in his new café in Cherry village. This is the only place babushkas can work in the village, so their reserve utility is 0. A babushka's utility function is $U_i = \sqrt{w_i} - \theta_i e_i$, where w_i is the wage, e_i is the level of effort (love and care) babushkas put into cakes, and θ_i is the type of babushkas (private information). It is known that the half of babushkas has $\theta_1 = 2$ and the other half has $\theta_2 = 4$. The more efforts are invested into cooking, the higher revenue the café gets. So uncle Vanya's profit is $\pi = 8e - w$.

Task

1) Find contracts $\{e, w\}$ offered by uncle Vanya to babushkas of different types if information is symmetric.
2) Show that the contracts found in the part 1 are not optimal if information is asymmetric.
3) Find contracts $\{e, w\}$ uncle Vanya offer to babushkas of different types in case of information asymmetry, when he uses screening to overcome the problem of adverse selection.

Solution

1) Find contracts $\{e, w\}$ offered by uncle Vanya to babushkas of different types if information is symmetric.

Since information is symmetric, uncle Vanya knows the concrete type of each babushka. So he offers each babushka the contract designed for her. In order to do this, uncle Vanya maximizes his profit $\pi(e_i, w_i)$ if *individual rationality constraint* (IR) of each babushka holds: the utility babushka gets in case of signing a contract should not be lower than the reserve utility.

$$\pi(e_i, w_i) = 8e_i - w_i \to \max_{e_i, w_i}$$

s.t. $U_i = \sqrt{w_i} - \theta_i e_i \geq 0$

Then we write down Lagrangian:

$$\mathcal{L} = 8e_i - w_i + \lambda_i(\sqrt{w_i} - \theta_i e_i)$$

First-order conditions:

$$\frac{\partial \mathcal{L}}{\partial e_i} = 8 - \lambda_i \theta_i = 0,$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = -1 + \frac{\lambda_i}{2\sqrt{w_i}} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = \sqrt{w_i} - \theta_i e_i = 0.$$

We get the following set of optimal contracts:

$$e_1 = 1, w_1 = 4,$$

$$e_2 = {}^1\!/_4, w_2 = 1.$$

Uncle Vanya gets the expected profit $E\pi = 2{,}5$.

2) Show that the contracts found in the part 1 are not optimal if information is asymmetric.

When information is asymmetric, uncle Vanya cannot distinguish between different types of babushkas. Hence, babushkas can use hidden (private) information about their type and choose the contract not designed to them in order to increase their utility.

Then we calculate babushka's utility when each of them signs a contract designed to the babushka of different type and find if it is profitable for them:

$U_1(e_2, w_2) = 1 - 0.5 = 0.5 > 0$=> babushka of type 1 will manipulate private information and choose the contract designed to babushka of type 2,
$U_2(e_1, w_1) = 2 - 4 = -2 < 0$=> babushka of type 2 will choose the contract designed to her type.

Uncle Vanya gets the expected profit $E\pi = 1$ that is lower than is the case of symmetric information.

3) Find contracts $\{e, w\}$ uncle Vanya offer to babushkas of different types in case of information asymmetry, when he uses screening to overcome the problem of adverse selection.
When uncle Vanya uses screening, he offers contracts that helps him to distinguish between babushka's types. So he should choose such contracts that provide incentives to babushkas of both types to choose only contract designed to them, not to babushkas of another type.
Hence, uncle Vanya takes into account not only *individual rationality constraint (IR)*, but also *incentive compatibility constraint (IC)*.
His problem is the following:

$$E\pi(e_1, w_1, e_2, w_2) = \frac{1}{2}(8e_1 + 8e_2 - w_1 - w_2) \to \max_{e_1, e_2, w_1, w_2}$$

s.t. $U_i = \sqrt{w_i} - \theta_i e_i \geq 0$ (IR)
$U_i(e_j, w_j) \geq U_j(e_i, w_i)$ (IC), $i \neq j$.

In more details:

$\sqrt{w_1} - 2e_1 \geq 0$ (IR1),
$\sqrt{w_2} - 4e_2 \geq 0$ (IR2),
$\sqrt{w_1} - 2e_1 \geq \sqrt{w_2} - 2e_2$ (IC1),
$\sqrt{w_2} - 4e_2 \geq \sqrt{w_1} - 4e_1$ (IC2),

IR2 is binding. Then IR1 is always true, so we can omit it.
IC1 is binding (no extra utility for type 1 when taking contract 1). IC2 is always true, because as we have shown earlier, there are no incentives for babushkas of type 2 to choose contract 1.
So the binding conditions are:

$\sqrt{w_2} - 4e_2 = 0$ (IR2),
$\sqrt{w_1} - 2e_1 = \sqrt{w_2} - 2e_2$ (IC1).

We express w_1 and w_2 from IR2 and IC1 and get the following:

$$w_1 = 4(e_1 + e_2)^2,$$
$$w_2 = 16e_2^2.$$

And then reformulate uncle Vanya's problem:

$$E\pi(e_1, w_1, e_2, w_2) = \frac{1}{2}(8e_1 + 8e_2 - 4(e_1 + e_2)^2 - 16e_2^2) \to \max_{e_1, e_2, w_1, w_2}$$

First-order conditions:

$$\frac{\partial E\pi}{\partial e_1} = 8 - 8(e_1 + e_2) = 0,$$
$$\frac{\partial E\pi}{\partial e_2} = -e_1 - e_2 + 1 - 4e_2 = 0.$$

We get the following set of optimal contracts when uncle Vanya uses screening:

$$e_1 = 1, w_1 = 4,$$
$$e_2 = 0, w_2 = 0.$$

Uncle Vanya gets the expected profit $E\pi = 2$ that is lower than is the case of symmetric information, but higher than in the case of adverse selection without screening.