Problem 2. Grossman-Hart Model

Problem Statement

Let us consider relationships of the two firms – the buyer and the seller. At some point in time #1 the buyer (Nike) creates an agreement with the seller (some Chinese factory) and signs a contract for the provision of basic t-shirts. At that moment Nike officials and Chinese representatives know that later, at a point in time #2, sewing pattern for this model of t-shirts might need improvement. However as they do not know yet what the improvement will be, they cannot include it in a basic contract. They will be still able to do it at the point in time #2 when they obtain this information.

Chinese factory's costs connected to changes in design of t-shirts are c = 10 m \$. Nike's additional profit $\hat{V}$ is $V = 20$ m \$ with a probability α (which means reconsideration of a contract is worth it), or $V = 0$ with a probability $(1-\alpha)$. Probability α is defined by the volume of Nike's specific investments I: $I = \gamma\alpha^2$ where $\gamma = 8$. Both sides are risk-neutral and have equal bargaining power: Nike's bargaining power is $\beta = 0{,}5$, and Chinese factory's power is $1-\beta = 0{,}5$. This means that if they start to bargain about anything (income, costs, etc.), they redistribute it equally.

Task

1. Consider the situation of the first best: Nike and Chinese factory act as one firm. Find an optimal level of $\alpha_1, I_1, E\pi_1, E\pi_1^b, E\pi_1^s$ (probability of a change in design, optimal level of investment, common expected payoff, buyer's expected payoff, seller's expected payoff).
2. Consider the situation where Nike and a Chinese factory act independently and bargain about the change of design. Both players might block decision on a change of design. Find $\alpha_2, I_2, E\pi_2, E\pi_2^b, E\pi_2^s$.
3. Let us suppose both sides decide to integrate, and the right of making final decision on a change of design is given to a Chinese factory. Find $\alpha_3, I_3, E\pi_3, E\pi_3^b, E\pi_3^s$.
4. Let us suppose the right of final decision belongs to Nike. Both sides integrate and pass the right of a final decision on a change of design to the buyer. During the second period there is a bargaining process on the change of design. Find $\alpha_4, I_4, E\pi_4, E\pi_4^b, E\pi_4^s$. What sum of money will the Chinese factory request from Nike for the right to make a final decision?

Solution

1. Both seller and buyer, acting together, are able to influence the probability of changing design, which in its turn might lead to extra profits. Both sides aim to increase common extra profits. Let us find the social optimum, which is the cumulative expected extra profit created when both sides maximize this value:

$$\max_{\alpha} \mathrm{E}\pi_1 = \alpha \cdot (V - c) + (1-\alpha) \cdot (0 - 0) - \gamma\alpha^2.$$

First-order condition:

$$\alpha_1 = \frac{V-c}{2\gamma} = \frac{20-10}{2 \cdot 8} = 0{,}625.$$

Than **the optimal volume of specific investments** at moment #1 is

$$I_1 = \gamma\alpha^2 = \frac{(V-c)^2}{4\gamma} = \frac{(20-10)^2}{4\cdot 8} = 3{,}125 \text{ m \$},$$

and the maximum of the total expected extra profit is

$$\mathrm{E}\pi_1 = \frac{(V-c)^2}{4\gamma} = 3{,}125 \text{ m \$},.$$

2. If players do not come to an agreement, change of design will not be implemented. At the point of time #2 Chinese factory and Nike bargain on extra profit of changes in design. Both of sides have equal bargaining power, so each side will expect a half of extra income (half of extra profit, and half of additional costs). Having predicted results of bargaining process, Nike (*buyer*), will maximize its expected extra profit

$$\max_{\alpha} \mathrm{E}\pi_2^b = \beta[\alpha\cdot(V-c)] - \gamma\alpha^2.$$

First-order condition:

$$\alpha_2 = \frac{\beta(V-c)}{2\gamma} = \frac{0{,}5(20-10)}{2\cdot 8} = 0{,}3125,$$

and **specific investments volume** for Nike is

$$I_2 = \frac{\beta^2(V-c)^2}{4\gamma} = \frac{0{,}5^2(20-10)^2}{4\cdot 8} = 0{,}78125 \text{ m \$},$$

expected extra profit for Nike is

$$\mathrm{E}\pi_2^b = \frac{\beta^2(V-c)^2}{4\gamma} = 0{,}78125 \text{ m \$},$$

and for the Chinese factory (*seller)* consequently is

$$\mathrm{E}\pi_2^s = \frac{\beta^2(V-c)^2}{2\gamma} = 1{,}5625 \text{ m \$}.$$

The total expected extra profit in case there is no integration is

$$\mathrm{E}\pi_2 = 2{,}34375 \text{ m \$},$$

the value of which is less than an optimal level of a total expected extra profit.

3. At the moment of time #2 Nike and Chinese factory will bargain on whether they should make changes in design or not, but the final decision will be given to the Chinese factory. If changes do not bring extra profit, the Chinese factory will not introduce changes in design; if they do, sides will bargain. As bargaining power is equal for both sides, each will have a half of extra profits.

Then, foreseeing this, in the moment of tome #1 Nike stems out of the supposition that its expected profit is half of the total expected profit to be gained from changing design. Nike will maximize this profit by trying to influence the probability of the positive outcome. Consequently, Nike is going to solve the same problem as the one described in the previous point.

4. At the moment #1 Nike pays a certain sum of money to a Chinese factory for the right to make decision on changing design. As Nike does not incur any expenses, it can insist on making changes in design in any case, even if $V = 0$. The Chinese factory, on the contrary, is interested in avoiding costs c that are connected to changing design of a product they make. Consequently, if changes in design bring extra profit ($V = 20$), the Chinese factory incurs expenses. If changes in design do not bring extra profit ($V = 0$), at the moment of time #2 Chinese factory will bargain with Nike for the possibility not to make changes in design.

In case at the moment of time #2 changes in design are connected to extra profit $V > 0$ for Nike, *status quo* becomes an effective strategy, and a reconsideration of a contract is not necessary. If the bargaining power of sides is equal, Nike will ask for a sum $c \cdot \xi = c/2$ for a permission not to introduce any changes in design.

In case the expected extra profit from changing design is $V = 0$, *status quo* does not appear to be an effective strategy, and it is worth reconsidering the contract. Than the optimal volume of Nike's specific investments is defined by the idea to maximize:

$$\max_{\alpha} \mathrm{E}\pi_4^b = \alpha V + (1-\alpha)c \cdot \xi - \gamma\alpha^2 .$$

First-order condition:

$$\alpha_4 = \frac{V - c\xi}{2\gamma} = 0{,}9375 ,$$

and the **optimal volume of Nike's specific investments** is

$$I_4 = \frac{(V - c\xi)^2}{4\gamma} = 7{,}03125 \text{ m \$},$$

Nike's expected extra profit is:

$$\mathrm{E}\pi_4^b = 12{,}03125 \text{ m \$},$$

Consequently, if the right of making a final decision on changing design belongs to Nike, it introduces specific investments that are larger than socially optimal. The expected extra profit of a Chinese factory (with no consideration of the initial compensation from Nike) is:

$\mathrm{E}\pi_4^s = -\alpha c - (1-\alpha)c\xi = -9{,}6875$ m \$, which is negative.

The total expected profit is:

$$\mathrm{E}\pi_4 = \mathrm{E}\pi_4^s + \mathrm{E}\pi_4^b = 2{,}34375 \text{ m \$}.$$

The Chinese factory will ask the following amount of money for the right of making decision on changing design

$$\mathrm{E}\pi_2^s - \mathrm{E}\pi_4^s = 11{,}25 \text{ m \$}.$$