Monitoring Costs

Problem Statement

Director of a ski resort has no time for a non-stop presence at work, so he hires a manager to control for the quality of services and check employees' work. Director fixes a salary for a manager - ***f*** rubles. If this manager does his or her job well and makes efforts ***e*** while solving ongoing problems at work, director receives an income of ***R*** rubles. If this manager does not perform his or her job well, overall quality of services falls, resort looses clients, and its income falls to zero. Being aware of such risks, director may monitor the work of the resort, which will cost ***i*** rubles. Director pays a salary both in case there is no monitoring, and in case the monitoring does not show any significant deviations from the manager's contract. Otherwise salary is not to be paid. Monitoring costs and monitoring results are not interdependent.

Task

1. Write down a pay-off matrix for this game describing this interrelation.
2. Find how parameters (***R, e, f, i***) should be related when a Nash equilibrium in pure strategies does not exist.
3. Find a Nash equilibrium in mixed strategies (using parameters found in p.2).
4. Which parameters influence the choice of a strategy by the manager in the Nash equilibrium in mixed strategies? And what about the strategy of the director? Show how the choice of equilibrium strategy varies with a change of one of the parameters.

Solution

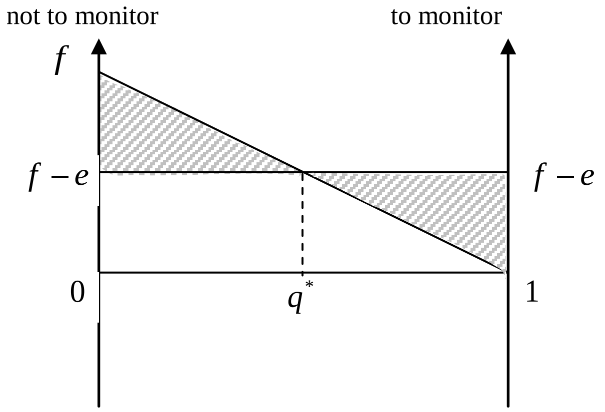
1. In general form the matrix of a game looks the following way:

		Director	
		monitors	*does not monitor*
Manager	*works well*	$f-e; R-f-i$	$f-e; R-f$
	does not work properly	$0; -i$	$f; -f$

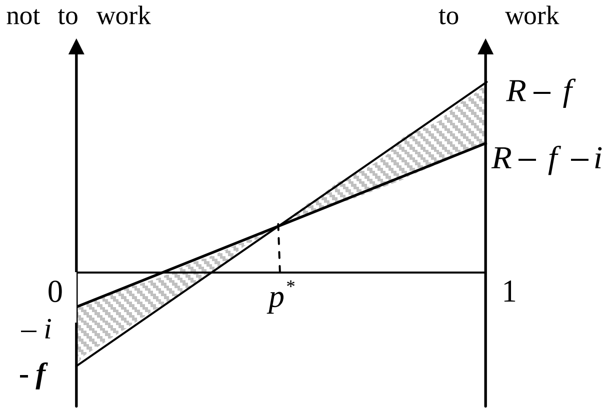
2. If director "monitors", the best strategy for a manager would be "to work", as benefits for the latter are larger in this case: $f-e>0$. If director "does not monitor", the best response of a manager is the strategy of "not working properly", because: $f > f-e$.

If manager chooses "to work", director will choose "not to monitor", because: $R-f > R-f-i$. If manager decides "not to work properly", in order to find the best solution director will need to check the relation between ***i*** and ***f***. On one hand, manager's salary (***f***) might be less than monitoring costs (***i***). Then director decides "not to monitor" manager's work, and as a result, a Nash equilibrium in pure strategies exists:– "not to monitor" and "not to work properly". But we have to show that the Nash equilibrium in pure strategies does not exist, as it is possible only if manager's salary is larger than monitoring costs: ***f*** > ***i***.

3. Let us draw the equilibrium in mixed strategies in a graphic way.



Picture 1. Director's behaviour



Picture 2. Manager's behaviour

Picture 1 shows the benefits of a manager for both cases of director's choice: "to monitor" and "not to monitor". Vertical axes show the benefits of a manager, and the horizontal axis shows possibility of choosing one or another strategy by director. If manager chooses "to work", the benefits are $(f - e)$ for both situations: when his or her employer "monitors", $q = 1$, and when his or her employer "does not monitor", $q = 0$. Let us draw the benefits of a strategy "to work" on corresponding axes, and connect them with a line. This line represents a combination of many expected benefits depending on the probability of director's choice of one strategy over another.

If manager chooses "not working well" his strategy, with a probability of $q = 1$ director will choose "to monitor", so manager will not get any benefits; and with a probability of $q = 0$ director will choose "not to monitor", so manager will get a benefit (f). Let us connect benefits of this strategy with a line.

We can see it on a picture that the two lines cross at one dot forming the two similar triangles. Stemming from properties of similar triangles, as the ratio of heights of similar triangles put on corresponding sides is equal to the relation of these sides, it is possible to find an equilibrium probability. Thereby we have:

$$\frac{q}{1-q} = \frac{f-(f-e)}{f-e} \quad \Rightarrow q = \frac{e}{f}$$

so the probability of director choosing "to monitor" depends on salary and efforts of his or her employee.

Following the same pattern, let us find the equilibrium probability for the manager's choice (p). As shown in picture 2, vertical axes depict benefits of employer in case manager chooses the "to work well" or the "not to work properly" strategy. The horizontal axis shows the probability of manager's choice for both strategies. If director chooses "to monitor", he receives a benefit of $(R - f)$ in a situation where manager "works well" ($p = 1$), and $(- i)$, while in a situation where manager "does not work well" – ($p = 0$). Let us draw the benefits of the strategy "to monitor" on corresponding axes, and connect them with a line.

If director chooses "not to monitor", with a probability of $p = 1$ he or she will benefit $(R - f - i)$, and with a probability of $p = 0$ he or she will get $(- f)$. In the same manner, let us draw the benefits of "not monitoring" on corresponding axes, and connect them with a line. The crossing of lines creates the equilibrium probability of manager's strategic choice.

$$\frac{p}{1-p} = \frac{-i-(-f)}{R-f-(R-f-i)} \quad \Rightarrow p = \frac{f-i}{f-i+i} = 1 - \frac{i}{f}$$

so the probability of manager choosing the strategy "to work" depends on salary and monitoring costs.

4. We found that probability of choice of one or another strategy by manager in Nash equilibrium in mixed strategies depends on salary and monitoring costs. Probability of choice of one or another strategy by director in Nash equilibrium in mixed strategies depends on manager's efforts and, again, on his or her salary. Optimal strategies do not depend on the scale of income director receives when manager works well. Employee's choice of strategy is defined by relation of his or her salary and transaction costs: the higher the monitoring costs are, the less stimuli for control there are for an employer, and the more beneficial it is for an employee to work worse, knowing about his or her impunity. Employer's decision on a scale of control is defined by relation of an employee' salary to costs connected with his attitude to work. The higher these costs are, the more control such employee requires.